Corrigé

1. $\left|\frac{3}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}\right| = \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{3\sqrt{3}}{2}\right)^2} = 3.$

   D'où $\cos(\alpha_1) = \frac{x_1}{|z_1|} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$ et $\sin(\alpha_1) = \frac{y_1}{|z_1|} = \frac{\frac{3\sqrt{3}}{2}}{3} = \frac{\sqrt{3}}{2}$.

   Ainsi, $\alpha_1 = \frac{\pi}{3} + k\times 2\pi, k\in\mathbb{Z}$ et donc $z_1 = 3\left[\cos\left(\frac{\pi}{3}\right)+\mathrm{i}\sin\left(\frac{\pi}{3}\right)\right]$.

2. $|\pi\mathrm{i}| = \pi$. De plus, z_2 est un imaginaire pur de partie imaginaire strictement positive, donc $\arg = \frac{\pi}{2}+k\times 2\pi, k\in\mathbb{Z}$. D'où $z_2 = \pi\left[\cos\left(\frac{\pi}{2}\right)+\mathrm{i}\sin\left(\frac{\pi}{2}\right)\right]$.

3. $|6+6\sqrt{3}\mathrm{i}| = \sqrt{6^2+\left(6\sqrt{3}\right)^2} = 12.$

   D'où $\cos(\alpha_3) = \frac{x_3}{|z_3|} = \frac{6}{12} = \frac{1}{2}$ et $\sin(\alpha_3) = \frac{y_3}{|z_3|} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$.

   Ainsi $\alpha_3 = \frac{\pi}{3}+k\times 2\pi, k\in\mathbb{Z}$. D'où $z_3 = 12\left[\cos\left(\frac{\pi}{3}\right)+\mathrm{i}\sin\left(\frac{\pi}{3}\right)\right]$.

4. $|-2+2\mathrm{i}| = \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}$. D'où $\cos(\alpha_3) = \frac{x_4}{|z_4|} = -\frac{2}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ et $\sin(\alpha_3) = \frac{y_4}{|z_4|} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$. Ainsi $\alpha_4 = \frac{3\pi}{4}+k\times 2\pi, k\in\mathbb{Z}$. Donc $z_4 = 2\sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right)+\mathrm{i}\sin\left(\frac{3\pi}{4}\right)\right]$.