Corrigé

Pour développer, on utilise les formules suivantes.

$$(a+b)^2 = a^2 + 2 \times a \times b + b^2 \qquad \text{(identité remarquable)}$$

$$(a-b)^2 = a^2 - 2 \times a \times b + b^2 \qquad \text{(identité remarquable)}$$

$$a \times (c+d) = a \times c + a \times d \qquad \text{(distributivité)}$$

$$(a+b) \times (c+d) = a \times c + a \times d + b \times c + b \times d \qquad \text{(double distributivité)}$$

1. Développer.

a)

$$\begin{aligned}(x+3)^2 &= x^2 + 2 \times 3 \times x + 3^2 \\ &= x^2 + 6x + 9\end{aligned}$$

b)

$$\begin{aligned}(2x+5)^2 &= (2x)^2 + 2 \times 5 \times (2x) + 5^2 \\ &= 4x^2 + 20x + 25\end{aligned}$$

c)

$$\begin{aligned}\left(\frac{x}{2} + \frac{4}{3}\right)^2 &= \left(\frac{x}{2}\right)^2 + 2 \times \frac{4}{3} \times \frac{x}{2} + \left(\frac{4}{3}\right)^2 \\ &= \frac{x^2}{4} + \frac{4}{3}x + \frac{16}{9}\end{aligned}$$

d)

$$\left(\frac{5}{7}x + \frac{1}{2}\right)^2 = \left(\frac{5}{7}x\right)^2 + 2 \times \frac{1}{2} \times \left(\frac{5}{7}x\right) + \left(\frac{1}{2}\right)^2$$

$$= \frac{25}{49}x^2 + \frac{5}{7}x + \frac{1}{4}$$

e)

$$\begin{aligned}\frac{2}{5}(3x+10)^2 &= \frac{2}{5}\left[(3x)^2 + 2\times 10\times(3x) + 10^2\right] \\ &= \frac{2}{5}\left(9x^2 + 60x + 100\right) \\ &= \frac{2}{5}\times 9x^2 + \frac{2}{5}\times 60x + \frac{2}{5}\times 100 \\ &= \frac{18}{5}x^2 + 24x + 40\end{aligned}$$

f)

$$(x-4)^2 = x^2 - 2\times 4\times x + 4^2$$

$$= x^2 - 8x + 16$$

g)

$$(3x-8)^2 = (3x)^2 - 2\times 8\times(3x) + 8^2$$

$$= 9x^2 - 48x + 64$$

h)

$$\begin{aligned}\left(\frac{x}{5} - \frac{3}{4}\right)^2 &= \left(\frac{x}{5}\right)^2 - 2\times\frac{3}{4}\times\frac{x}{5} + \left(\frac{3}{4}\right)^2 \\ &= \frac{x^2}{25} - \frac{3}{10}x + \frac{9}{16}\end{aligned}$$

i)

$$\begin{aligned}\left(\frac{9}{4}x - \frac{3}{2}\right)^2 &= \left(\frac{9}{4}x\right)^2 - 2\times\frac{9}{4}\times\left(\frac{3}{2}x\right) + \left(\frac{3}{2}\right)^2 \\ &= \frac{81}{16}x^2 - \frac{27}{4}x + \frac{9}{4}\end{aligned}$$

j)

$$\frac{6}{7}(5x-1)^2 = \frac{6}{7}\left[(5x)^2 - 2\times 1\times (5x) + 1^2\right]$$
$$= \frac{6}{7}\left(25x^2 - 10x + 1\right)$$
$$= \frac{6}{7}\times 25x^2 - \frac{6}{7}\times 10x + \frac{6}{7}\times 1$$
$$= \frac{150}{7}x^2 - \frac{60}{7}x + \frac{6}{7}$$

k)

$$(-x+6)^2 = (-x)^2 + 2\times 6\times (-x) + 6^2$$
$$= x^2 - 12x + 36$$

l)

$$(-3x+2)^2 = (-3x)^2 + 2\times 2\times (-3x) + 2^2$$
$$= 9x^2 - 12x + 4$$

m)

$$\left(-\frac{x}{3}+\frac{7}{5}\right)^2 = \left(-\frac{x}{3}\right)^2 + 2\times\frac{7}{5}\times\left(-\frac{x}{3}\right) + \left(\frac{7}{5}\right)^2$$
$$= \frac{x^2}{9} - \frac{14}{5}x + \frac{49}{25}$$

n)

$$\left(-\frac{2}{9}x+\frac{4}{7}\right)^2 = \left(-\frac{2}{9}x\right)^2 + 2\times\frac{4}{7}\times\left(-\frac{2}{9}x\right) + \left(\frac{4}{7}\right)^2$$
$$= \frac{4}{81}x^2 - \frac{16}{63}x + \frac{16}{49}$$

2. Développer.

a)

$$7 \times (2x+3) = 7 \times (2x) + 7 \times 3$$

$$= 14x + 21$$

b)

$$-3(4x-5) = (-3) \times (4x) + (-3) \times (-5)$$

$$= -12x + 15$$

c)

$$9(-4x+1) = 9 \times (-4x) + 9 \times 1$$

$$= -36x + 9$$

d)

$$3x(7x-6) = (3x) \times (7x) + (3x) \times (-6)$$

$$= 21x^2 - 18x$$

e)

$$-5x(-4x+3) = (-5x) \times (-4x) + (-5x) \times 3$$

$$= 20x^2 - 15x$$

f)

$$(5x+4)(6x+3) = (5x) \times (6x) + (5x) \times 3 + 4 \times (6x) + 4 \times 3$$

$$= 30x^2 + 15x + 24x + 12$$

$$= 30x^2 + 39x + 12$$

g)

$$(6x-7)(2x+8) = (6x) \times (2x) + (6x) \times 8 - 7 \times (2x) - 7 \times 8$$

$$= 12x^2 + 48x - 14x - 56$$

$$= 12x^2 + 34x - 56$$

h)

$$(7x+2)(4x-3) = (7x) \times (4x) + (7x) \times (-3) + 2 \times (4x) + 2 \times (-3)$$

$$= 28x^2 - 21x + 8x - 6$$

$$= 28x^2 - 13x - 6$$

i)

$$(-6x+1)(3x-3) = (-6x) \times (3x) + (-6x) \times (-3) + 1 \times (3x) + 1 \times (-3)$$

$$= -18x^2 + 18x + 3x - 3$$

$$= -18x^2 + 21x - 3$$

j)

$$(-4x+6)(-5x-9) = (-4x)\times(-5x) + (-4x)\times(-9) + 6\times(-5x) + 6\times(-9)$$
$$= 20x^2 + 36x - 30x - 54$$
$$= 20x^2 + 6x - 54$$

k)

$$(7x+2)(4x-3) = (7x)\times(4x) + (7x)\times(-3) + 2\times(4x) + 2\times(-3)$$
$$= 28x^2 - 21x + 8x - 6$$
$$= 28x^2 - 13x - 6$$

l)

$$\left(\frac{2}{9}x+\frac{5}{2}\right)\left(\frac{4}{3}x-6\right) = \left(\frac{2}{9}x\right)\times\left(\frac{4}{3}x\right) + \left(\frac{2}{9}x\right)\times(-6) + \frac{5}{2}\times\left(\frac{4}{3}x\right) + \frac{5}{2}\times(-6)$$
$$= \frac{8}{27}x^2 - \frac{4}{3}x + \frac{10}{3}x - 15$$
$$= \frac{8}{27}x^2 + 2x - 15$$

m)

$$\left(-\frac{7}{3}x+\frac{2}{9}\right)\left(-\frac{6}{5}x-\frac{1}{2}\right) = \left(-\frac{7}{3}x\right)\times\left(-\frac{6}{5}x\right) + \left(-\frac{7}{3}x\right)\times\left(-\frac{1}{2}\right) + \frac{2}{9}\times\left(-\frac{6}{5}x\right) + \frac{2}{9}\times\left(-\frac{1}{2}\right)$$
$$= \frac{14}{5}x^2 + \frac{7}{6}x - \frac{4}{15}x - \frac{1}{9}$$
$$= \frac{14}{5}x^2 + \frac{9}{10}x - \frac{1}{9}$$

3. Développer.

a)

$$\begin{aligned}(5x-3)^2-(7x+1)^2 &= \left[(5x)^2-2\times 3\times(5x)+3^2\right]-\left[(7x)^2+2\times 1\times(7x)+1^2\right]\\ &= \left(25x^2-30x+9\right)-\left(49x^2+14x+1\right)\\ &= 25x^2-30x+9-49x^2-14x-1\\ &= -24x^2-44x+8\end{aligned}$$

b)

$$\begin{aligned}(-4x+2)^2-(3x-2)^2 &= \left[(-4x)^2+2\times 2\times(-4x)+2^2\right]-\left[(3x)^2-2\times 2\times(3x)+2^2\right]\\ &= \left(16x^2-16x+4\right)-\left(9x^2-12x+4\right)\\ &= 16x^2-16x+4-9x^2+12x-4\\ &= 7x^2-4x\end{aligned}$$

c)

$$\begin{aligned}-2(6x+5)^2-4(3x-8)^2 &= -2\left[(6x)^2+2\times 5\times(6x)+5^2\right]-4\left[(3x)^2-2\times 8\times(3x)+8^2\right]\\ &= -2\left(36x^2+60x+25\right)-4\left(9x^2-48x+64\right)\\ &= -72x^2-120x-50-36x^2+192x-256\\ &= -108x^2+72x-306\end{aligned}$$

d)

$$\begin{aligned}(6x+7)^2-3\left(8x^2-4x+2\right) &= \left[(6x)^2+2\times 7\times(6x)+7^2\right]-24x^2+12x-6\\ &= 36x^2+84x+49-24x^2+12x-6\\ &= 12x^2+96x+43\end{aligned}$$

e)

$$(5x+7)^2-(-3x+2)(x-6)=\left[(5x)^2+2\times 7\times(5x)+7^2\right]-[(-3x)\times x+(-3x)\times(-6)+2\times x+2\times(-6)]$$

$$=\left(25x^2+70x+49\right)-\left(-3x^2+20x-12\right)$$

$$=25x^2+70x+49+3x^2-20x+12$$

$$=28x^2+50x+61$$

f)

$$-2(7x-4)^2-(7x-4)(-3x-9)$$

$$=-2\left[(7x)^2-2\times 4\times(7x)+4^2\right]$$

$$-[(7x)\times(-3x)+(7x)\times(-9)-4\times(-3x)+(-4)\times(-9)]$$

$$=-2\left[49x^2-56x+16\right]-\left[-21x^2-63x+12x+36\right]$$

$$=-2\left[49x^2-56x+16\right]-\left[-21x^2-51x+36\right]$$

$$=-98x^2+112x-32+21x^2+51x-36$$

$$=-77x^2+163x-68$$

g)

$$(x+6)(3x-2)-(7x+4)(-2x+8)$$

$$=[x\times(3x)+x\times(-2)+6\times(3x)+6\times(-2)]$$

$$-[(7x)\times(-2x)+(7x)\times 8+4\times(-2x)+4\times 8)$$

$$=\left(3x^2-2x+18x-12\right)-\left(-14x^2+56x-8x+32\right)$$

$$=\left(3x^2+16x-12\right)-\left(-14x^2+48x+32\right)$$

$$=3x^2+16x-12+14x^2-48x-32$$

$$=17x^2-32x-44$$

h)

$$-4(3x-8)(-2x+3)-5(7x+3)(-6x+9)$$

$$= -4\left[(3x)\times(-2x)+(3x)\times 3-8\times(-2x)-8\times 3\right]$$

$$\qquad -5\left[(7x)\times(-6x)+(7x)\times 9+3\times(-6x)+3\times 9\right)$$

$$= -4\left(-6x^2+9x+16x-24\right)-5\left(-42x^2+63x-18x+27\right)$$

$$= -4\left(-6x^2+25x-24\right)-5\left(-42x^2+55x+27\right)$$

$$= 24x^2-100x+96+210x^2-275x-135$$

$$= 234x^2-375x-39$$