

Preuve

$$f(x) = ax^2 + bx + c$$

$$= a \left[x^2 + \frac{b}{a}x + \frac{c}{a} \right]$$

$$= a \left[x^2 + 2 \times \frac{b}{2a}x + \frac{c}{a} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{(b^2 - 4ac)}{4a^2} \right]$$

$$= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{\Delta}{4a^2} \right] \quad \text{avec } \Delta = b^2 - 4ac$$

$$= a \left(x - \frac{-b}{2a} \right)^2 - \frac{\Delta}{4a}$$